

Origami Voronoi Diagrams

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Abstract

This paper describes a class of origami built from Voronoi diagrams that contain folded gussets proportional to the distances between cell sites. Described here is a general formula for gussets of all possible sizes and how the intersections between gussets resolve so that all the folded gussets sit orthogonal to the Voronoi plane. Furthermore, many of the Voronoi operations relate directly to origami fold operations and this paper describes how to solve Voronoi diagrams entirely by hand on a material decorated with points.

The edges of the solved Voronoi diagram are discoverable simply by performing origami axiom #2 on the Voronoi cell sites. However, it's important to know where to begin and end each line, so this paper briefly introduces the reader to the Voronoi's dual, Delaunay triangulation, how to spot difficult triangulations (large empty spaces, near-vicinity intersections), and how it's possible to discover their circumcenters with just origami.

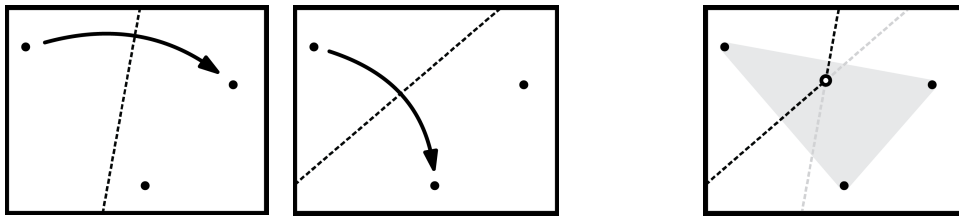


Figure 1: *the Delaunay triangle circumcenter, discoverable with two folds of axiom #2*

Once the voronoi edges are creased, one additional step is required to uncover the gusset edges and joints. These edges are geometric replicas of each Voronoi cell but with an affine scale between 0 and 1 that places the homothetic center on the cell's dot. This paper describes how to fold by hand gussets that relate to an affine scale of $1/2$ or $1/4$, and provides the generalized form of gusset formation at any scale. Another delightful symmetry arises if you perform Kawasaki's theorem on each intersection, due to the constraints of a Voronoi graph, 3 creases are guaranteed which comprise the gusset joints but also each point in the direction of the 3 cell sites.

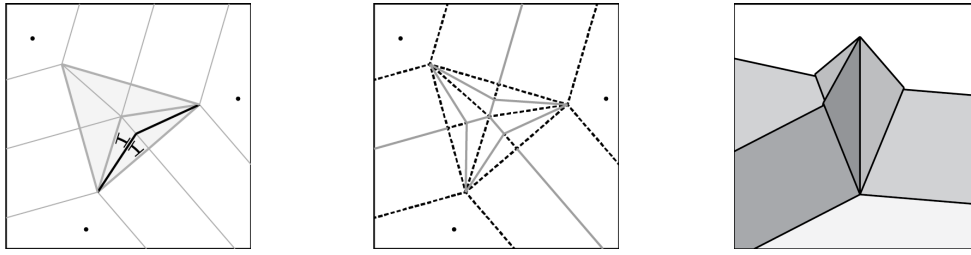


Figure 3: *Solving a simple gusset joint*

A gusset joint is *solved* when the 3 collapsed gussets lie upright orthogonal to the material plane and the 3 flaps' intersection is a line that is a normal vector to the plane. The simple gusset joint case occurs when the gusset triangle is acute. A right triangle case is also considered simple. Gusset joints become *complex* when the triangles become obtuse as the crimp opposite of the largest interior angle becomes non-existent. The solution involves absorbing the crimp angle it would have been in the calculation of the angles in the other 2 crimps.

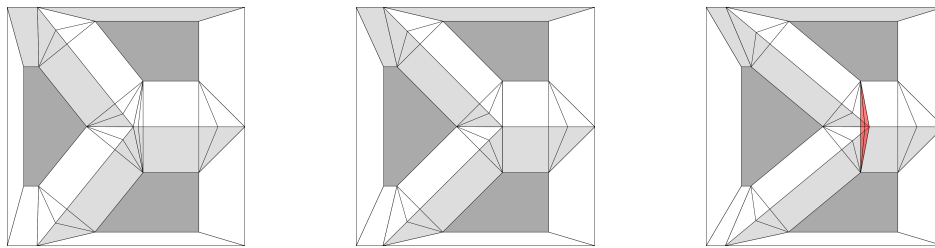


Figure 4: *Gusset joint moves from simple to complex*

A complex case arises when 2 gusset joints' triangles lie on top of one another. The solution requires casting rays and calculating intersection points with other crease lines to determine the proper length of the crease.

This method joins the Voronoi cell faces together to lie in the same plane, resulting in a gaussian curvature of 0. This paper outlines the rules discussed and what happens when you break them, one example being what happens when cell-scale is varied within the same Voronoi diagram. This creates non-zero curvature, outside of the scope of this paper, and nicely segues into the work discussed by Tomohiro Tachi and Erik Demaine in "Origamizer: A Practical Algorithm for Folding Any Polyhedron"

There is a working implementation of the algorithm written in javascript and accessible over the internet, descriptions and a link will be provided.

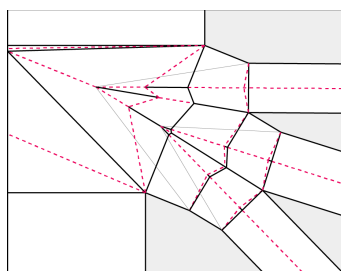


Figure 5: *Solved complex overlapping gusset joints*



Figure 6: *Folded voronoi*